MATH SPEAK - TO BE UNDERSTOOD AND MEMORIZED

1) **INEQUALITY** = a mathematical statement that contains one of these four ***inequality signs***: $<, \leq, >, \geq$. These ***inequality signs*** indicate that two expressions have different values.

REVIEW OF INEQUALITIES

I) **AN *INEQUALITY* IS A MATHEMATICAL STATEMENT CONTAINING ONE OF THESE FOUR SIGNS:** $<, \leq, >, \geq$ **LOCATED BETWEEN TWO DIFFERENT EXPRESSIONS.** ***Inequalities*** always contain one of these four signs: $<, \leq, >, \geq$, which indicates that the expression on one side of the ***inequality sign*** is a different value, either greater (larger) than or less (smaller) than, the expression on the other side.

e.g. INEQUALITY	MEANING (LS = Left Side; RS = Right Side)
$y < 3x - 2$	LS < RS; or the LS expression is less (smaller) than the RS expression.
$y > \frac{^-4}{5}x + 3$	LS > RS; or the LS expression is greater (bigger) than the RS expression.
$y \leq {}^-x$	LS $\leq$ RS; or the LS expression is equal to or less (smaller) than the RS expression.
$y \geq 5x - 3$	LS $\geq$ RS; or the LS expression is equal to or greater (bigger) than the RS expression.

II) ***INEQUALITIES* IN ONE VARIABLE**

A) ***Inequalities*** in one variable have only one kind of variable. The examples below are ***inequalities*** in one variable. Study them carefully: **NOTICE** that they all have only one kind of variable. Be sure you understand and memorize how to recognize them: **e.g.** $x > 3$; $x \leq {}^-4$; $2 > y$; $w \geq 9.5$; $a < 3.25$; $2x - 3 \leq 5x$

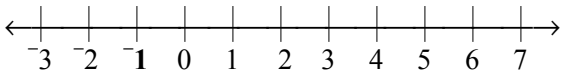
B) Since an ***inequality*** has one side that is a greater or smaller value than the other side, it has many possible solutions that are best illustrated with a graph. Since ***inequalities*** in one variable have only one type of variable, they are graphed on one ***AXIS***, the number line.

GRAPHING INEQUALITIES

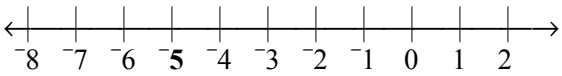
I) ***Inequalities*** are solved by the same process that is used to solve equations with one additional rule: ***WHEN AN INEQUALITY IS MULTIPLIED OR DIVIDED BY A NEGATIVE NUMBER THE INEQUALITY SIGN*** $(<, \leq, >, \geq)$ ***IS FLIPPED AROUND***: **i.e.** $<$ becomes $>$; $\geq$ becomes $\leq$.

A) ***SAMPLE PROBLEMS 1:*** Study these examples carefully. Be sure you understand and memorize the process used to complete them.

1) Solve $2x - 3 \geq {}^-5$



2) Solve $4 - 3x > 19$



3) Solve $^-3 < \frac{^-x}{5} - 2$

B) ***REQUIRED PRACTICE 1:*** Solve and graph these ***inequalities***. **SHOW THE PROCESS!!** **{Answers are on page 6 of these notes.}**

1) $3x+2\leq 11$ 2) $^{-}3x-3>9$ 3) $5\geq {}^{-}3x+11$ 4) $\frac{2}{3}x-5<{}^{-}9$ 5) $4-\frac{3}{2}x\leq 10$ 6) $4>\frac{-4}{5}x+2$

MATH SPEAK - TO BE UNDERSTOOD AND MEMORIZED

1) **SET** = a group of things.

2) **REAL NUMBERS** = $\mathbb{R}$ = the ***set*** of ***numbers*** that can be placed on a ***number line***.

3) **NATURAL NUMBERS** = $\mathbb{N}=\{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12,\ 13,\ \cdots\}$; ***natural numbers*** are composed of the ***set*** of positive ***entire/counting numbers***.

4) **WHOLE NUMBERS** = $W=\{0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12,\ 13,\ \cdots\}$; ***whole numbers*** are composed of the ***set*** of non-negative ***entire/counting numbers***.

5) **INTEGERS** = $I=\{\cdots,\ {}^{-}10,\ {}^{-}9,\ {}^{-}8,\ {}^{-}7,\ {}^{-}6,\ {}^{-}5,\ {}^{-}4,\ {}^{-}3,\ {}^{-}2,\ {}^{-}1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ \cdots\}$; ***integers*** are composed of the ***set*** of the negative and the non-negative ***entire/counting numbers***.

THE REAL NUMBER SYSTEM

I) ***REAL NUMBERS ARE NUMBERS THAT CAN BE PLACED ON A NUMBER LINE***. There are four ***sets (groups)*** of ***real numbers*** you must be familiar with, each of which is described in the pages below.

A) **REAL NUMBERS** are represented by the symbol ***R*** and are composed of all ***numbers*** that can be converted to decimals and placed on a ***number line***.

i.e. $\mathbb{R}={}^{-}1.0467,\ {}^{-}0.\overline{3},\ {}^{-}0.03,\ 0,\ 0.25,\ \frac{1}{2},\ 0.7505,\ 1,\ 2.5,\ \sqrt{7},\ \pi,\ \frac{11}{3}$

1) Within the ***set*** of ***Real numbers*** are three other ***sets*** you must know.

B) **NATURAL NUMBERS** are represented by the symbol ***N*** and are composed of the positive ***entire/counting numbers***.

i.e. $\mathbb{N}=$

1) ***Natural numbers*** are often called ***counting numbers*** because they are most often used to count the ***number*** of objects a person is interested in.

C) **WHOLE NUMBERS** are represented by the symbol ***W*** and are composed of the non-negative ***entire/counting numbers***. This means that ***whole numbers*** consist of the ***numbers*** 0 (zero), and all of the ***entire/counting numbers***.

i.e. $W=$

1) **NOTICE** that the ***set*** of ***whole numbers*** includes the ***set*** of ***natural numbers***, ***N***.

D) **INTEGERS** are represented by the symbol ***I***, and are composed of 0 (zero), and all the positive and negative ***counting/entire numbers***.

i.e. $I=$

1) **NOTICE** that the ***set*** of ***integers*** includes the ***set*** of ***whole numbers*** and the ***set*** of ***natural numbers***.

E) ***REQUIRED PRACTICE 2:*** Answer these questions. **{Answers are on page 6 of these notes.}**

1) Identify the ***sets*** (***natural numbers***, ***whole numbers*** and ***integers***) to which these ***numbers*** belong. A ***number*** may belong to one or more than one ***set***.

   a) 3 b) $^{-}105$ c) 0 d) $^{-}55$ e) $^{-}84$ f) 924

2) Name the ***set*** or ***sets*** of ***numbers*** that are found within the ***whole numbers***.

3) Name the ***set*** or ***sets*** of ***numbers*** that are found within the ***integers***.

4) Name the ***set*** or ***sets*** of ***numbers*** that are found within the ***real numbers***.

5) Which ***number*** is a ***whole number*** but not a ***natural number***?

6) Which ***number*** is an ***integer*** and a ***whole number*** but not a ***natural number***?

7) Which ***integer*** is neither positive nor negative?

II) **RELATING FOUR DIFFERENT SETS OF *REAL NUMBERS* TO EACH OTHER**

A) There are three different types of ***real numbers*** you must know***: natural numbers, whole numbers*** and ***integers***. The relationship between the three ***sets*** of ***real numbers*** is described below in two ways – **FIRST:** is a box diagram found directly below and **SECOND:** is a flow chart on page 8. The two descriptions can help you identify all the ***sets*** (groups) to which a ***number*** belongs.

1) **FIRST:** This box diagram describes how the four ***sets*** of ***real numbers*** you must know are related to each other. This diagram shows that some ***sets*** are part of other ***sets***: **e.g.** the ***set*** of ***natural numbers*** is part of the ***set*** of ***whole numbers*** as indicated by its box being inside the box containing the ***set*** of ***whole numbers***.

REAL NUMBERS = $\mathbb{R}$ = all ***numbers*** that can be placed on a ***number line***.

e.g. $^{-}1.0467,\ ^{-}0.\overline{3},\ ^{-}0.03,\ 0,\ 0.25,\ \frac{1}{2},\ 0.7505,\ 1,\ 2.5,\ \sqrt{7},\ \pi,\ \frac{11}{3}$

The ***set*** of ***real numbers*** includes all smaller ***sets*** within this box: ***decimal numbers***, ***fractions***, ***radical numbers***, ***powers***, ***integers***, ***whole numbers*** and ***natural numbers***.

INTEGERS = I = zero, 0, and all positive and negative ***counting/entire numbers***.

e.g. $I = \cdots\ ^{-}5,\ ^{-}4,\ ^{-}3,\ ^{-}2,\ ^{-}1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5\ \cdots$

The ***set*** of ***integers*** includes all smaller ***sets*** within this box: ***whole numbers*** and ***natural numbers***.

WHOLE NUMBERS = W = zero, 0, and all positive ***counting/entire numbers***.

e.g. $W = 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12\ \cdots$

The ***set*** of ***whole numbers*** includes all smaller ***sets*** within this box: ***natural numbers***.

NATURAL NUMBERS = $\mathbb{N}$ = all positive ***counting/entire numbers***.

e.g. $\mathbb{N} = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12\ \cdots$

2) **SECOND:** This flow chart describes how the 4 ***sets*** of ***real numbers*** you must know are related to each other.

Real numbers = $\mathbb{R}$ = all ***numbers*** that can be placed on a ***number line***.

e.g. $^{-}1.0467,\ ^{-}0.\overline{3},\ ^{-}0.03,\ 0,\ 0.25,\ \frac{1}{2},\ 0.7505,\ 1,\ 2.5,\ \sqrt{7},\ \pi,\ \frac{11}{3}$

The ***set*** of ***real numbers*** includes all smaller ***sets*** under this box: the ***integers***, ***whole numbers***, and ***natural numbers***.

Integers = I = zero, 0, and all positive and negative ***counting/entire numbers***.

e.g. I = ... ,$^{-}5$, $^{-}4$, $^{-}3$, $^{-}2$, $^{-}1$, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...

The ***set*** of ***integers*** includes all smaller ***sets*** under this box: the ***whole numbers*** and ***natural numbers***.

Whole numbers = W = zero, 0, and all positive ***counting/entire numbers***.

e.g. W = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...

The ***set*** of ***whole numbers*** includes all smaller ***sets*** under this box: the ***natural numbers***.

Natural numbers = $\mathbb{N}$ = all positive ***counting/entire numbers***.

e.g. N = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...

B) **WEB RESOURCES:** These websites may help you understand the ***real number system***.

1) http://jamesbrennan.org/algebra Click on the link titled: **The Real Number System**

MATH SPEAK - TO BE UNDERSTOOD AND MEMORIZED

1) **SET NOTATION** = A method of defining the variables, equation or inequality and domain and or range of a relation.
2) **DOMAIN** = the input numbers, the values of the horizontal axis that are part of the graph.
3) **RANGE** = the output numbers, the values of the vertical axis that are part of the graph.
4) **RESTRICTION** = limitations to the domain (input numbers) and range (output numbers) of the given equation or inequality.

INTRODUCING SET NOTATION

I) ***Set notation*** is a method of identifying the variables, equation or inequality, and the ***domain*** and/or ***range*** used to draw a graph of a ***relation***. ***Set notation*** is a method mathematicians use to restrict the ***domain*** (input numbers) and ***range*** (output numbers) of a given equation or inequality.

A) The two statements given below are written in ***set notation***. Be sure you understand and memorize what each part of ***set notation*** means.

e.g. $\{x \mid x > 6\ , x \in \mathbb{R}\}$ This statement is written in ***set notation*** and means the variable in the math statement is an ***x***, the math statement is the inequality $x > 6$ $x \succ 6$ and the values of ***x*** that are used to draw its graph are $x \in \mathbb{R}$ (all real numbers). Since $x \in \mathbb{R}$, the inequality does not have any restricted values.

e.g. $\{(x, y) \mid y = 3x + 4\ , x \in \mathrm{I}, y \in \mathrm{I}\}$ This statement is written in ***set notation*** and means the variables in the math statement are ***x*** and ***y***, the math statement is the equation $y = 3x + 4$, the ***domain*** is $x \in \mathrm{I}$ (Integers) and the ***range*** is $y \in \mathrm{I}$ (Integers). Since $x \in \mathrm{I}$ and $y \in \mathrm{I}$, the ***domain*** and ***range*** are restricted to positive and negative entire numbers.

GRAPHING MATH STATEMENTS WRITTEN IN SET NOTATION

I) **REMEMBER**: ***Set notation*** gives you information you need to set up the appropriate grid and ***restrict*** the ***domain*** and ***range*** in order to draw the graph of the ***relation***.

A) **USE THESE STEPS TO DRAW THE GRAPH AN EQUATION OR INEQUALITY WRITTEN IN *SET NOTATION***

STEP 1: ***Identify the variable or variables to determine which type of graph to draw.***

a) If one variable, use a single horizontal number line.

b) If two variables, use a grid with horizontal and vertical axes.

STEP 2: ***Identify the restrictions.***

STEP 3: ***Draw the graph using the appropriate method and the given restrictions.***

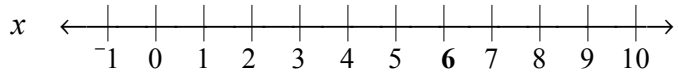
1) ***<u>SAMPLE PROBLEM 3:</u>*** Study these examples carefully. Be sure you understand and memorize the process used to complete them. **INSTRUCTIONS:** Graph the solution set for these inequalities.

a) $\{x \mid x > 6\ , x \in \mathbb{R}\}$

1: ***Identify the variable or variables to determine which type of graph to draw.***

2: ***Identify the restrictions.***

3: ***Draw the graph using the appropriate method and the given restrictions.***



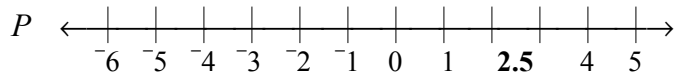
b) $\{P \mid P \leq 2.5\ , P \in \mathrm{I}\}$

1: ***Identify the variable or variables to determine which type of graph to draw.***

2: ***Identify the restrictions.***

Continued on the next page.

3: ***Draw the graph using the appropriate method and the given restrictions.***

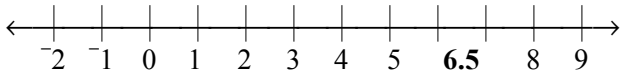


c) $\{m \mid m \leq 6.5\,,\, m \in \mathrm{W}\}$

1: ***Identify the variable or variables to determine which type of graph to draw.***

2: ***Identify the restrictions.***

3: ***Draw the graph using the appropriate method and the given restrictions.***



2) ***REQUIRED PRACTICE 3:*** Graph the ***solution set*** for these inequalities. **SHOW THE PROCESS!!**
{Answers are on page 6 of these notes.}

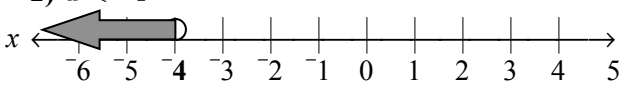
1) $\{x \mid x \leq 5\,,\, x \in \mathbb{R}\}$ 2) $\{a \mid a > {}^{-}7\,,\, a \in \mathrm{I}\}$ 3) $\{z \mid z > {}^{-}7\,,\, z \in \mathrm{W}\}$ 4) $\{d \mid d \leq 5.5\,,\, d \in \mathbb{N}\}$

ANSWERS TO THE REQUIRED PRACTICE

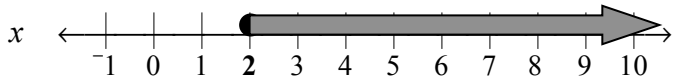
Required Practice 1 from page 2

1) $x \leq 3$

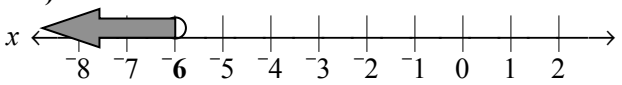
2) $x < {}^{-}4$



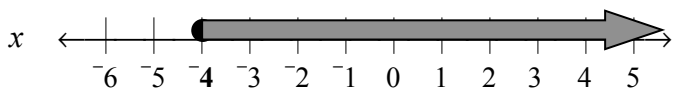
3) $x \geq 2$



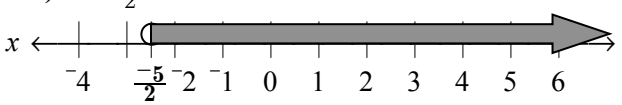
4) $x < {}^{-}6$



5) $x \geq {}^{-}4$



6) $x > \frac{{}^{-}5}{2}$

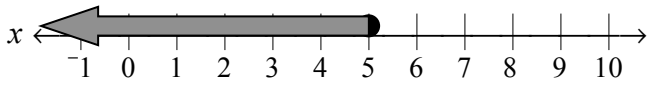


Required Practice 2 from page 3

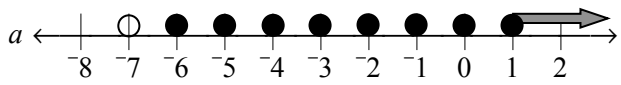
1a) $\mathbb{N}$, W & I **1b)** I **1c)** W & I **1d)** I **1e)** I **1f)** $\mathbb{N}$, W & I **2)** 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
3) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 **2)** ***Natural numbers*** **3)** ***Natural*** & ***Whole numbers*** **4)** ***Integers***, ***Naturals*** & ***Whole numbers*** **5)** 0 **6)** 0 **7)** 0

Required Practice 3 from page 6

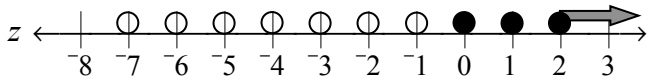
1) $\{x \mid x \leq 5, x \in \mathbb{R}\}$



2) $\{a \mid a > {}^{-}7, a \in \mathrm{I}\}$



3) $\{z \mid z > {}^{-}7, z \in \mathrm{W}\}$



4) $\{d \mid d \leq 5.5, d \in \mathbb{N}\}$