MATH SPEAK - TO BE UNDERSTOOD AND MEMORIZED

1) **ALGEBRA** = the branch of mathematics that simplifies math statements that contain ***variables***.

2) **VARIABLE** = a letter that represents an unknown number. **e.g.** $a,\ b,\ x,\ y$

3) **TERM** = a mathematical statement consisting of a number, or a combination of a number and one or more ***variables***.

   e.g. $3,\ 71,\ \frac{3}{4},\ x,\ x^2,\ {}^{-}5x,\ 4x^3y,\ \frac{x}{4},\ \frac{{}^{-}x}{4},\ \frac{3x}{5},\ \frac{2x-6}{5x},\ \frac{3x^2-x+1}{8x-9}$

4) **CONSTANT TERM** = a mathematical statement consisting of a number. **e.g.** $3,\ 71,\ \frac{3}{4},\ {}^{-}54$

5) **COEFFICIENT** = a number multiplying one or more ***variables***.

e.g. ***Term*:**	x	${}^{-}x$	$3xy$	$\frac{1}{3}x^2y^5$	$\frac{{}^{-}5}{9}p^3$
***Coefficient*:**	1	${}^{-}1$	3	$\frac{1}{3}$	$\frac{{}^{-}5}{9}$

6) **LIKE TERMS** = ***terms*** that have the same power (***variable*** raised to the same exponent).

   e.g. $x,\ {}^{-}5x,\ \frac{4}{7}x$ and ${}^{-}0.253x$ are ***like terms***

   e.g. $xy^2,\ {}^{-}3xy^2,\ 5xy^2,\ 2004.7xy^2$ and $\frac{8}{13}xy^2$ are ***like terms***

7) **UNLIKE TERMS** = ***terms*** that have **different** powers (different ***variable(s)*** or the same ***variable(s)*** raised to different ***exponents***).

   e.g. $x,\ {}^{-}5x^2,\ \frac{4}{7}x^6$ and ${}^{-}0.253x^3$ are ***unlike terms*** because the ***exponents*** are different

   e.g. $xy^2,\ {}^{-}3py^2,\ 5xn^2,\ 2004.7ay^2$ and $\frac{8}{13}cd^2$ are ***unlike terms*** because the ***variables*** are different

8) **ALGEBRAIC EXPRESSION** = **POLYNOMIAL** = an ***expression*** composed of numbers and ***variables***.

   e.g. $x,\ 7x-4,\ x^2+3x-2,\ 8x^3-21x^2+43x$

9) **EQUATION** = a mathematical statement indicating that two ***expressions*** are equal to each other..

   e.g. $x=3,\ 7x-4=5,\ x^2+3x-2=3x-4,\ 8x^3-21x^2+43x=0$

10) **MONOMIAL** = an ***algebraic expression*** composed of a single ***term*** having at least one ***variable***.

   e.g. $x,\ x^2,\ {}^{-}5x,\ 4x^3y,\ \frac{x}{4},\ \frac{{}^{-}x}{4},\ \frac{3x}{5},\ \frac{2x-6}{5x},\ \frac{3x^2-x+1}{8x-9}$

11) **BINOMIAL** = an ***algebraic expression*** composed of **two different *terms***, at least one of which has a ***variable***, separated by addition (+) or subtraction (–) signs.

   e.g. $x+1,\ x^2-4,\ (3x+2y),\ \frac{9}{5}x-\frac{7}{9}x^3$

12) **TRINOMIAL** = an ***algebraic expression*** composed of **three different *terms***, at least one of which has a ***variable***, separated by addition (+) or subtraction (–) signs.

   e.g. $x^2+2x-3,\ x-y+4,\ \frac{1}{6}x^4-x^3+21$

13) **POLYNOMIAL** = an ***algebraic expression*** composed of **four or more different *terms***, at least one of which has a ***variable***, separated by addition (+) or subtraction (–) signs.

   e.g. $x^3-x^2+2x-3,\ x-y+3z+4,\ \frac{1}{6}x^4-x^3+0.4x-21$

14) **DISTRIBUTION** = **EXPAND** = a process of multiplying a single ***term*** to each ***term*** of an ***algebraic expression*** that is inside a set of brackets.

   e.g. $6(x+y)=6x+6y,\ 7x(4x+3y-6)=28x^2+21xy-42x$

15) **FOIL** = a set of instructions used to multiply two ***binomials***.

SIMPLIFYING ALGEBRAIC EXPRESSIONS

I) ***ALGEBRA*** is the branch of mathematics that works with ***VARIABLES***. ***A VARIABLE is a letter that represents an unknown number:*** **e.g.** $a,\ b,\ x,\ y$.

A) The purpose of ***algebra*** is to simplify ***ALGEBRAIC EXPRESSIONS***. ***ALGEBRAIC EXPRESSIONS are expressions that contain terms having numbers and one or more variables***.

e.g. $x,\ \ 7x-4,\ \ x^2+3x-2,\ \ 8x^3-21x^2+43x$

B) ***Algebraic expressions are composed of a single term, or two or more terms that are separated by addition and subtraction signs. TERMS are composed of a number, or a combination of a number and one or more variables.*** Each of these examples is a single ***term***.

e.g. $3,\ 71,\ \frac{3}{4},\ x,\ x^2,\ {}^{-}5x,\ 4x^3y,\ \frac{x}{4},\ \frac{{}^{-}x}{4},\ \frac{3x}{5},\ \frac{2x-6}{5x},\ \frac{3x^2-x+1}{8x-9}$

1) ***Terms*** are the building blocks of ***algebraic expressions***, as a result it is extremely important that you understand what a ***term*** is and are able to identify individual ***terms***. ***Terms*** are added or subtracted from one another resulting in different kinds of ***algebraic expressions***. Study these examples carefully as they will help you identify the different ***terms*** within mathematical statements.

a) $x,\ x^2,\ {}^{-}5x,\ 4x^3y,\ \frac{x}{4},\ \frac{{}^{-}x}{4},\ \frac{3x}{5}$ are ***algebraic expressions*** that consist of only one ***term*** (each expression lacks an addition or subtraction sign) thus they are called ***monomials***.

b) $\frac{2x-6}{5x},\ \frac{3x^2-x+1}{8x-9}$ are ***algebraic expressions*** that consist of only one ***term*** thus they are called ***monomials***. These examples can be confusing because they are fractions that contain more than one ***term*** within the ***denominator***, the ***numerator***, or both the ***denominator*** and ***numerator***. It is important, however, to **REMEMBER** that each is a single ***fraction***, which makes each a single ***term*** regardless of what its ***denominator*** and ***numerator*** are composed of.

c) $7x-4,\ \ x^2-2,\ \ 7x-\frac{3x}{5},\ \ 21x^2+43x,\ \ \frac{2x-6}{5x}+4x,\ \ {}^{-}6x^3+25x,\ \ {}^{-}9x^4+\frac{3x^2-x+1}{8x-9}$ are ***algebraic expressions*** that consist of two ***terms*** separated by an addition (+) or a subtraction (–) sign thus they are called ***binomials***. **REMEMBER:** $\frac{2x-6}{5x}$ and $\frac{3x^2-x+1}{8x-9}$ are single ***fractions*** and which means each is a single ***term***. A ***fraction*** is a single ***term*** even if its ***numerator*** and or ***denominator*** are composed of more than one ***term***.

d) $x^2+7x-4,\ x^2+x-2,\ {}^{-}x^3-7x^2+\frac{3x}{5},\ \frac{2x-6}{5x}+4x-85,\ {}^{-}6x^3+25x+2,\ {}^{-}9x^4+\frac{3x^2-x+1}{8x-9}+1$ are ***algebraic expressions*** that consist of three ***terms*** separated by an addition (+) or a subtraction (–) sign thus they are called ***trinomials***. **REMEMBER:** $\frac{2x-6}{5x}$ and $\frac{3x^2-x+1}{8x-9}$ are single ***fractions*** and which means each is a single ***term***. A fraction is a single ***term*** even if its ***numerator*** and or ***denominator*** are composed of more than one ***term***.

C) ***REQUIRED PRACTICE 1:*** Identify these ***algebraic expressions*** as a ***monomial***, ***binomial*** or ***trinomial***. **{Answers are on page 9 of these notes.}**

1) $3x^2+5x$ 2) $6x^2-\frac{3}{5x}$ 3) $\frac{5-x}{x}-3x-5$ 4) $\frac{5-x}{x}-\frac{{}^{-}x+5}{2}-\frac{4x-7}{4x}$

II) **ADDITION AND SUBTRACTION WITHIN *ALGEBRAIC EXPRESSIONS* CAN ONLY OCCUR BETWEEN *LIKE TERMS*.** When combining ***like terms*** by addition or subtraction, combine the ***coefficients*** only, **DO NOT CHANGE THE POWERS (VARIABLES RAISED TO AN EXPONENT) IN ANY WAY**.

A) ***SAMPLE PROBLEMS 6:*** Study these examples carefully. Be sure you understand and memorize the process used to complete them. **Instructions:** Simplify these ***algebraic expressions*** then classify the results as a ***monomial***, ***binomial, trinomial*** or ***polynomial***.

1) $4x+6x$ **2)** $3x-x+4-9$ **3)** $2x^2-5x^2-x$

4) $2x^2+1-3x^2+x$ **5)** $^-3x^3+x^2-4x^3+3x^2-1$ **6)** $3x^2-2xy+5y^2-5xy-3y^2$

B) ***REQUIRED PRACTICE 2:*** Simplify these ***algebraic expressions*** then classify the results as a ***monomial***, ***binomial*** or ***trinomial***. **SHOW THE PROCESS!!** **{Answers are on page 9 of these notes.}**

1) $3x^2+5x+2x^2-6x$ 2) $6x^2-3-7x^2+10$ 3) $3xy+5y-8y+6xy$

4) $4x^2y-5x^2+x^2y-2x^2$ 5) $5x^2y-3xy^2-2x^2y+4xy^2$ 6) $3y^3+7y+6y^3-8y+12y^2$

III) **DISTRIBUTION IS DEFINED HERE**: **i.e.** $a(b+c)=ab+ac$ ← ***MEMORIZE THIS PATTERN!!***

A) ***Distribution*** is also known as ***expanding*** and is used to eliminate of a set of brackets containing an ***algebraic expression*** when a ***negative sign or a term is directly in front of the brackets***. The set of brackets are eliminated by multiplying each ***term*** of the ***algebraic expression*** inside the set of brackets by the ***negative sign or the term directly in front of (touching) the brackets***.

B) ***SAMPLE PROBLEMS 3:*** Study these examples carefully. Be sure you understand and memorize the process used to complete them. **Instructions:** ***Expand*** these ***algebraic expressions*** then classify them as a ***monomial***, ***binomial, trinomial*** or ***polynomial***.

1) $5(2x+5)$ **2)** $^-3(^-5x+2)$ **3)** $^-4(3x^2+2x-3)$

4) $^-(x^2-4x)+7$ **5)** $3x-(4x^3-8x^2+9)+1$ **6)** $^-x^6-(^-3-x^8+x^3)+15x$

C) ***REQUIRED PRACTICE 3:*** Simplify these ***algebraic expressions*** then classify the results as a ***monomial***, ***binomial, trinomial*** or ***polynomial***. **SHOW THE PROCESS!!** **{Answers are on page 9 of these notes.}**

1) $^-3(5x-4)$ 2) $2(5x-3)$ 3) $^-3(^-2a^2+5a-7)$ 4) $^-6(^-x+7)$

5) $^-4(3x-7)+2$ 6) $^-(5x-3)$ 7) $^-(4a^2+3a-9)$ 8) $^-(^-2x^2-8x-27)-13$

9) $^-3-(6x-5)+9$ 10) $^-(5x^2-3x)+x$ 11) $7-(9a+3a^3)-9$ 12) $5x^7-(^-4x^5-x^9-7)-15$

IV) **MULTIPLICATION OF *ALGEBRAIC EXPRESSIONS* CAN OCCUR BETWEEN LIKE TERMS OR UNLIKE TERMS.** When multiplying terms, the ***coefficients are multiplied and the exponents of each variable are added***.

A) ***SAMPLE PROBLEMS 4:*** Study these examples carefully. Be sure you understand and memorize the process used to complete them. **Instructions:** Simplify these ***expressions***.

1) $2x^4 \bullet 3x^6$ **2)** $(^-2x)(^-5x^3)$ **3)** $(3xy)(^-5x^2y)(2x^3y)$

B) ***REQUIRED PRACTICE 4:*** Simplify these ***expressions***. **SHOW THE PROCESS!!** **{Answers are on page 7 of these notes.}**

1) $7x^2 \bullet 3x^5$ 2) $^-2x \times 4x^2$ 3) $(15xy^3)(3x^4y)$

4) $(^-4x^2y^2)(^-xy^5)$ 5) $(^-2x^3y^2)(^-3x^3y^6) \times {}^-6xy$ 6) $(^-8x^5y^3z)(5xz^8)(^-4x^7y^4z^9)$

V) ***FOIL* IS A SET OF INSTRUCTIONS USED TO EXPAND (MULTIPLY) TWO *BINOMIAL* EXPRESSIONS TOGETHER.** Each letter represents multiplication of a specific pair of terms within the two ***binomials***. ***FOIL* is extremely important!** Be sure you understand and memorize it.

F = multiply the **F**irst terms of each binomial together.
O = multiply the **O**utside terms of each binomial together.
I = multiply the **I**nside terms of each binomial together.
L = multiply the **L**ast terms of each binomial together.

A) ***SAMPLE PROBLEMS 5:*** Solve these problems. Be sure you understand and memorize the process used to complete them. **NOTICE** how the process of ***foiling*** is used to complete each problem. **Instructions:** Expand these ***expressions***.

1) $(x+2)(x+3)$ **2)** $(2x-7)(2x-7)$

B) ***REQUIRED PRACTICE 5:*** Expand these ***expressions***. **SHOW YOUR PROCESS!!** **{Answers are on page 7 of these notes.}**

1) $(x+2)(x-3)$ 2) $(x-4)(x-5)$ 3) $(2x+1)(x-3)$ 4) $(2x+5)(3x+4)$

C) ***SAMPLE PROBLEMS 6:*** Solve these problems. Be sure you understand and memorize the process used to complete them. **NOTICE** how the process of ***foiling*** is used to complete each problem. **Instructions:** Expand these ***expressions***.

1) $(x+2)^2$ **2)** $(8x-3)^2$

D) ***REQUIRED PRACTICE 6:*** Expand these ***expressions***. **SHOW YOUR PROCESS!!** **{Answers are on page 6 of these notes.}**

1) $(x+4)^2$ 2) $(4x+5)^2$ 3) $(3x-5)^2$ 4) $(7x-6)^2$

MATH SPEAK - TO BE UNDERSTOOD AND MEMORIZED

1) **FACTORS** = numbers or variables multiplied together to create a term.

FACTORS

I) ***FACTORS ARE NUMBERS OR VARIABLES THAT ARE MULTIPLIED TOGETHER TO CREATE TERMS OR POLYNOMIALS***

A) The term $3x$ was created by multiplying 3 and *x* together; this means its ***factors*** are 3 and *x*. The term $^{-}7x^2y$ was created by multiplying $^{-}7$, *x*, *x* and *y* together. This means its ***factors*** are $^{-}7$, *x*, *x* and *y*. Because ***factors*** are used to make larger numbers, they are part of larger numbers.

II) **PRIME FACTORS**

A) ***PRIME NUMBERS are numbers that can only be divided by one and themselves to create entire numbers.*** Here are some examples of ***prime numbers***: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41. ***Prime numbers*** are useful to know because they can be used to find ***factors*** of larger numbers.

B) ***Prime factors*** are ***factors*** of large numbers that can only be divided by 1 and themselves to create entire numbers. Tree diagrams are used to find ***prime factors*** of larger numbers. To create tree diagrams, divide numbers by 5 if they end in 0 or 5; divide by 2 if they are even; if they are odd divide by 3, 7, 11, 13, etc. Tree diagram are useful because they show all the ***prime factors*** of the larger number. The ***prime factors*** can then be used to find other ***factors*** of the large number.

1) ***SAMPLE PROBLEM 7:*** Study these examples carefully. Be sure you understand and memorize the process used to complete them.

a) What are the ***prime factors*** of 75?

The ***prime factors*** of 75 are.

b) What are the ***prime factors*** of 24?

The ***prime factors*** of 24 are.

c) What are the ***prime factors*** of 147?

The ***prime factors*** of 147 are $3 \bullet 7 \bullet 7$.

d) What are the ***prime factors*** of 105?

The ***prime factors*** of 105 are $5 \bullet 3 \bullet 7$.

2) ***REQUIRED PRACTICE 7:*** State the ***prime factors*** of these numbers. **SHOW THE PROCESS!!** **{Answers are on page 7 of these notes.}**

a) 66 b) 125 c) 98 d) 162 e) 56

ANSWERS TO THE REQUIRED PRACTICE

Required Practice 1 from page 2

1) binomial **2)** binomial **3)** trinomial **4)** trinomial

Required Practice 2 from pages 2 & 3

1) $5x^2 - x$, binomial **2)** $^{-}x^2 + 7$, binomial **3)** $9xy - 3y$, binomial **4)** $5x^2y - 7x^2$, binomial

5) $3x^2y + xy^2$, binomial **6)** $9y^3 + 12y^2 - y$, trinomial

Required Practice 3 from page 3

1) $^{-}15x + 12$, binomial **2)** $10x - 6$, binomial **3)** $6a^2 - 15a + 21$, trinomial **4)** $6x - 42$, binomial

5) $^{-}12x + 30$, binomial **6)** $^{-}5x + 3$, binomial **7)** $^{-}4a^2 - 3a + 9$, trinomial **8)** $2x^2 + 8x + 14$, trinomial

9) $^{-}6x + 11$, binomial **10)** $^{-}5x^2 + 4x$, binomial **11)** $^{-}3a^3 - 9a - 2$, trinomial **12)** $x^9 + 5x^7 + 4x^5 - 8$, polynomial

Required Practice 4 from page 4

1) $21x^7$ **2)** $^{-}8x^3$ **3)** $45x^5y^4$ **4)** $4x^3y^7$ **5)** $^{-}36x^7y^9$ **6)** $160x^{13}y^7z^{18}$

Required Practice 5 from page 4

1) $x^2 - x - 6$ **2)** $x^2 - 9x + 20$ **3)** $2x^2 - 5x - 3$ **4)** $6x^2 + 23x + 20$

Required Practice 6 from page 5

1) $x^2 + 8x + 16$ **2)** $16x^2 + 40x + 25$ **3)** $9x^2 - 30x + 25$ **4)** $49x^2 - 84x + 36$

Required Practice 7 from page 5

a) $2 \bullet 3 \bullet 11$ **b)** $5 \bullet 5 \bullet 5$ **c)** $2 \bullet 7 \bullet 7$ **d)** $2 \bullet 3 \bullet 3 \bullet 3 \bullet 3$ **e)** $2 \bullet 2 \bullet 2 \bullet 7$
